Generation of Maximally Entangled Field-States in Cavities

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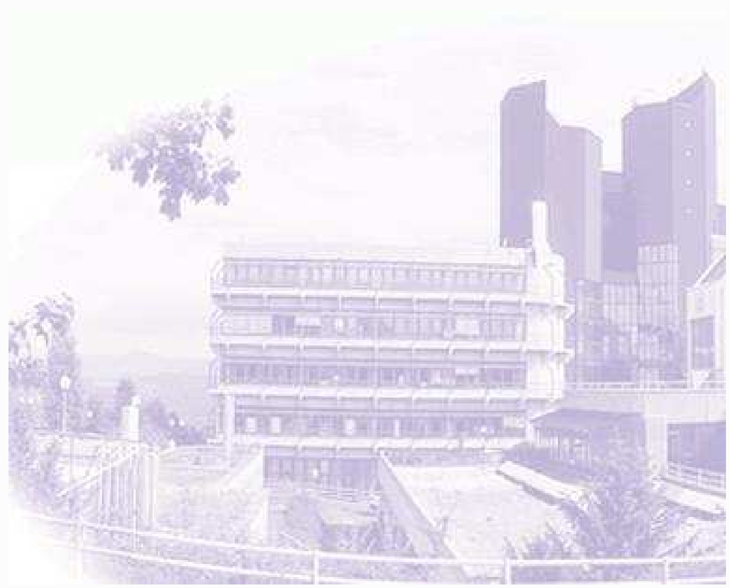

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Generation of Maximally Entangled Field-States in Cavities

Overview

- **Two-mode JCM**
- **Algebraic solution**
- **Entanglement between atoms and field modes**
- **Generation of entangled N-photon field states**
- **Applications**
  - Quantum Lithography
  - Quantum Teleportation

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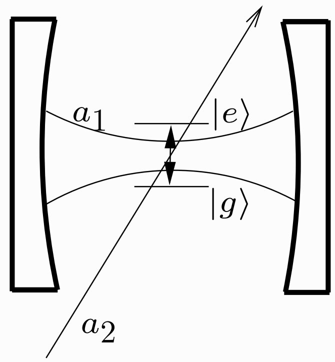
CQED Experiments for Generation of Entanglement

e.g. $|\Psi_N^{\pm}\rangle = \frac{1}{\sqrt{2}}\left(|N,0\rangle \pm |0,N\rangle\right)$



Ref.: S.M. Dutra and P.L. Knight, Phys. Rev. A **49**, 1506 (1994).

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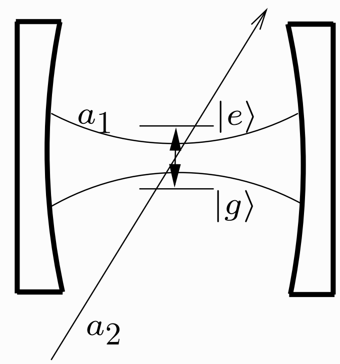

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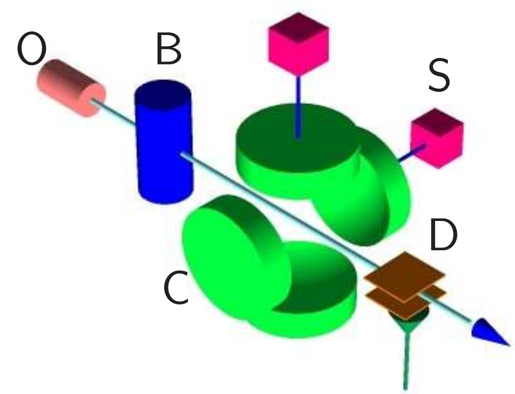

Proposal for a two-mode resonator experiment

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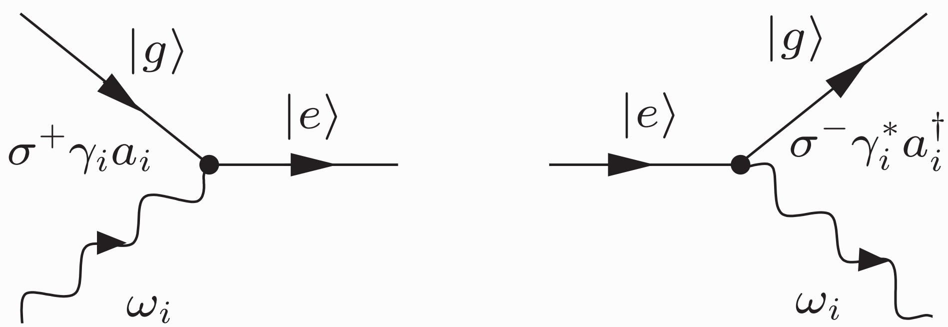
Two-Mode-JCM

$$H = \hbar\Omega\frac{\sigma_z}{2} + \hbar\omega_1 a_1^\dagger a_1 + \hbar\omega_2 a_2^\dagger a_2 + \hbar g\left[\sigma^+(\gamma_1 a_1 + \gamma_2 a_2) + \sigma^-(\gamma_1^* a_1^\dagger + \gamma_2^* a_2^\dagger)\right] \quad .$$

$$\sigma_z := |e\rangle\langle e| - |g\rangle\langle g| \ , \ \sigma^+ := |e\rangle\langle g| \ , \ \sigma^- := |g\rangle\langle e| \ , \ \mathbf{1} = |e\rangle\langle e| + |g\rangle\langle g|$$

$$\gamma_i = g_i/g \quad , \quad g = \sqrt{|g_1|^2 + |g_2|^2}$$



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Two-Mode-JCM

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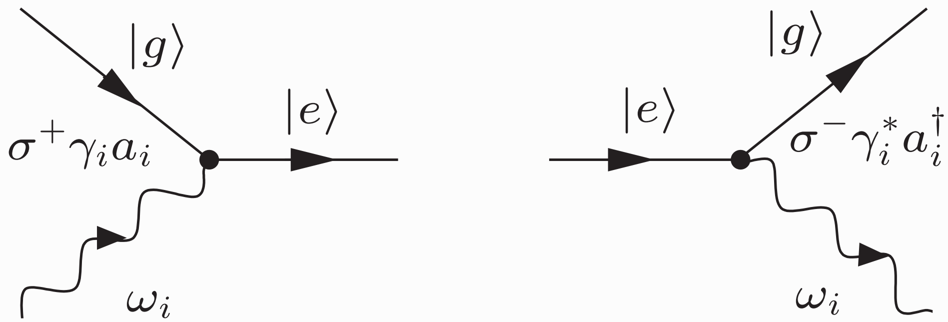

$\omega_1 = \omega_2 \quad , \quad \Delta = \Omega - \omega$:

$$H_0 = \hbar\omega\left[\frac{\sigma_z}{2} + a_1^\dagger a_1 + a_2^\dagger a_2\right] \quad ,$$

$$H_{\text{int}} = \hbar\left[\Delta\frac{\sigma_z}{2} + g\sigma^+(\gamma_1 a_1 + \gamma_2 a_2) + g\sigma^-(\gamma_1^* a_1^\dagger + \gamma_2^* a_2^\dagger)\right] \quad .$$

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Algebraic Solution

$$H_0 = \hbar\omega\left[\frac{\sigma_z}{2} + a_1^\dagger a_1 + a_2^\dagger a_2\right] \quad ,$$
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By introducing *quasimode* operators

$$A_1 = \gamma_1 a_1 + \gamma_2 a_2 \quad ,$$
$$A_2 = -\gamma_2^* a_1 + \gamma_1^* a_2 \quad ,$$

the interaction operator is reduced to an effective one-mode interaction operator.

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Algebraic Solution

$$
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\end{aligned}
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Algebraic Solution ($\Delta = 0$)

Calculation of the time evolution operator in the interaction picture:

$$U(t) = \mathrm{e}^{-\mathrm{i}H_{\mathrm{int}}t/\hbar}$$

$$U = U_{ee}|e\rangle\langle e| + U_{ge}|g\rangle\langle e| + U_{eg}|e\rangle\langle g| + U_{gg}|g\rangle\langle g| \quad .$$

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$$U_{ee} = \cos(\tau\sqrt{A_1^\dagger A_1 + 1}), \quad U_{ge} = A_1^\dagger \frac{\sin(\tau\sqrt{A_1^\dagger A_1 + 1})}{\mathrm{i}\sqrt{A_1^\dagger A_1 + 1}} \quad ,$$

$$U_{eg} = \frac{\sin(\tau\sqrt{A_1^\dagger A_1 + 1})}{\mathrm{i}\sqrt{A_1^\dagger A_1 + 1}} A_1, \quad U_{gg} = \cos(\tau\sqrt{A_1^\dagger A_1}) \quad , \tau := gt$$

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For $\Delta \neq 0$ changes are in terms of $\delta := \frac{\Delta}{2g}$,

e.g.

$$U_{ee} = \cos(\tau\sqrt{A_1^\dagger A_1 + 1 + \delta^2}) + \delta \frac{\sin(\tau\sqrt{A_1^\dagger A_1 + 1 + \delta^2})}{\sqrt{A_1^\dagger A_1 + 1 + \delta^2}} \quad .$$

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Algebraic Solution

How do these matrix elements act on two-mode Fock states $\{|n_1, n_2\rangle\}$?

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Algebraic Solution

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Definition of Fock states

$$|n_1, n_2\rangle := \frac{(a_1^\dagger)^{n_1}(a_2^\dagger)^{n_2}}{\sqrt{n_1!n_2!}}|0, 0\rangle \quad ,$$

and quasimode Fock states ($|0, 0\rangle = |0, 0\rangle\rangle$):

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Use Schwinger's oscillator model, where $j = (n_1 + n_2)/2$, $m = (n_1 - n_2)/2$:

$$|j,m\rangle\!\rangle_{\mathrm{S}} = \sum_{m'=-j}^{j} D^{(j)}_{m',m}(\varphi, \vartheta, \chi)\, |j, m'\rangle_{\mathrm{S}} \quad ,$$

$$|j,m\rangle_{\mathrm{S}} = \sum_{m'=-j}^{j} D^{(j)\dagger}_{m',m}(\varphi, \vartheta, \chi)\, |j, m'\rangle\!\rangle_{\mathrm{S}} \quad .$$

$$\gamma_i = |\gamma_i| \exp(\mathrm{i}\varphi_i), \quad \varphi = \varphi_1 - \varphi_2, \quad \chi = \varphi_1 + \varphi_2,$$

$$\cos(\vartheta/2) = |\gamma_1|, \quad \sin(\vartheta/2) = |\gamma_2|.$$

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Ref.: C. Wildfeuer and D.H. Schiller, Phys. Rev. A **67**, 053801 (2003).

Generation of entanglement between atoms and modes

Time evolution of $|g;\Phi_{\mathrm{F}}\rangle = \sum_{j=0}^{\infty}{}' \sum_{m=-j}^{j} \tilde{\tilde{b}}_{j,m}\, |g;j,m\rangle\rangle_{\mathrm{S}}$

$$
\begin{aligned}
|\Psi(t)\rangle \quad = \quad & \sum_{j=0}^{\infty}{}' \sum_{m=-j}^{j} \tilde{\tilde{b}}_{j,m} \cos(\tau\sqrt{j+m})\, |g;j,m\rangle\rangle_{\mathrm{S}} \\
& - \mathrm{i} \sum_{j=1/2}^{\infty}{}' \sum_{m=-j}^{j} \tilde{\tilde{b}}_{j,m} \sin(\tau\sqrt{j+m})\, \left|e; j-\tfrac{1}{2}, m-\tfrac{1}{2}\right\rangle\rangle_{\mathrm{S}} \quad .
\end{aligned}
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Example: (One mode case published by: J. Gea-Banacloche, Phys. Rev. Lett. **65**, 3385 (1990))

$$|\Phi_{\mathrm{F}}(0)\rangle = |\eta,0\rangle = \mathrm{e}^{-\frac{|\eta|^2}{2}} \sum_{n_1=0}^{\infty} \frac{\eta^{n_1}}{\sqrt{n_1!}} |n_1,0\rangle \quad .$$

Verifying the expansion coefficients $\tilde{\tilde{b}}_{j,m}$

$$|\eta,0\rangle = \mathrm{e}^{-|\eta|^2/2} \sum_{j=0}^{\infty}{}' \sum_{m=-j}^{j} \frac{\eta^{2j}}{\sqrt{(2j)!}} D^{(j)*}_{j,m}\, |j,m\rangle\rangle_{\mathrm{S}} \quad .$$

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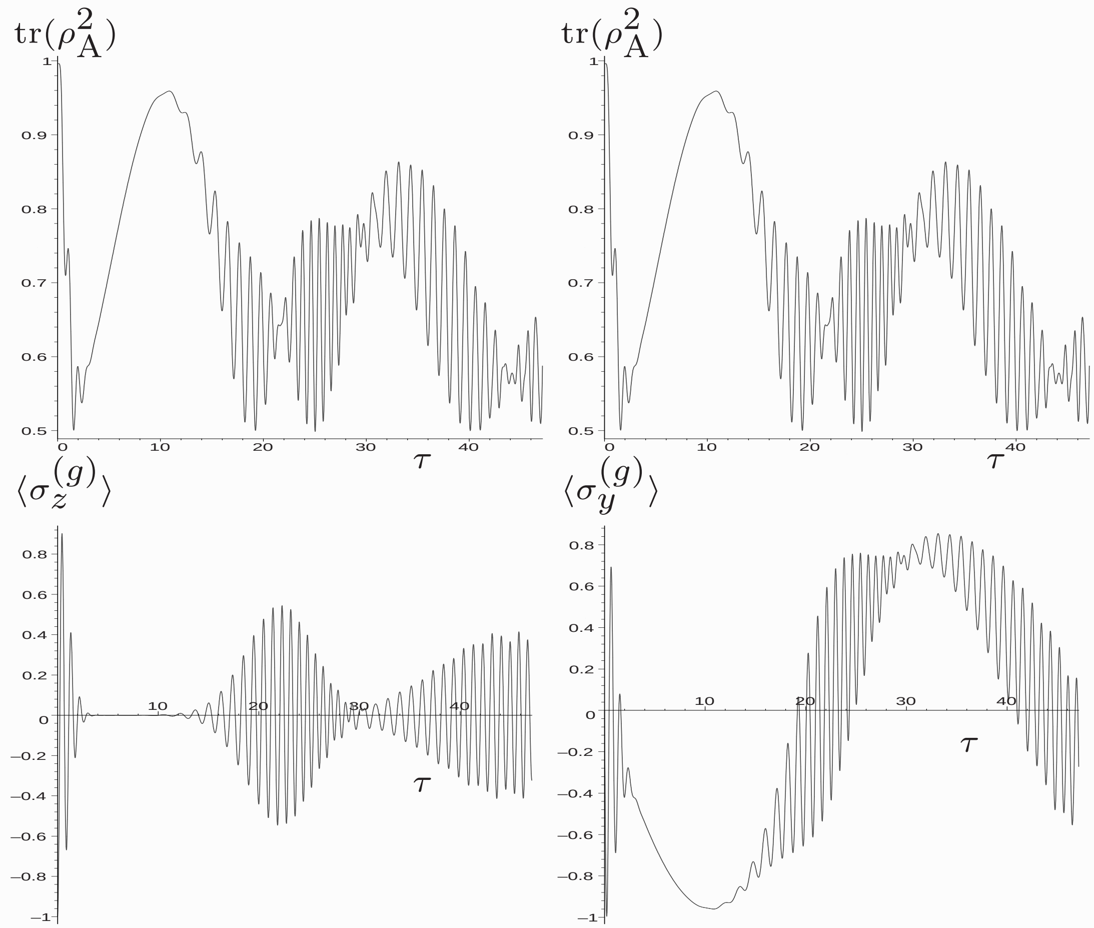
Generation of entanglement between atoms and modes

Parameter: Initial state $|g;\eta,0\rangle$, $g_1 = g_2$, $\overline{n} = 25$, $\eta = \sqrt{\overline{n}}\exp(-\mathrm{i}\varphi_1)$



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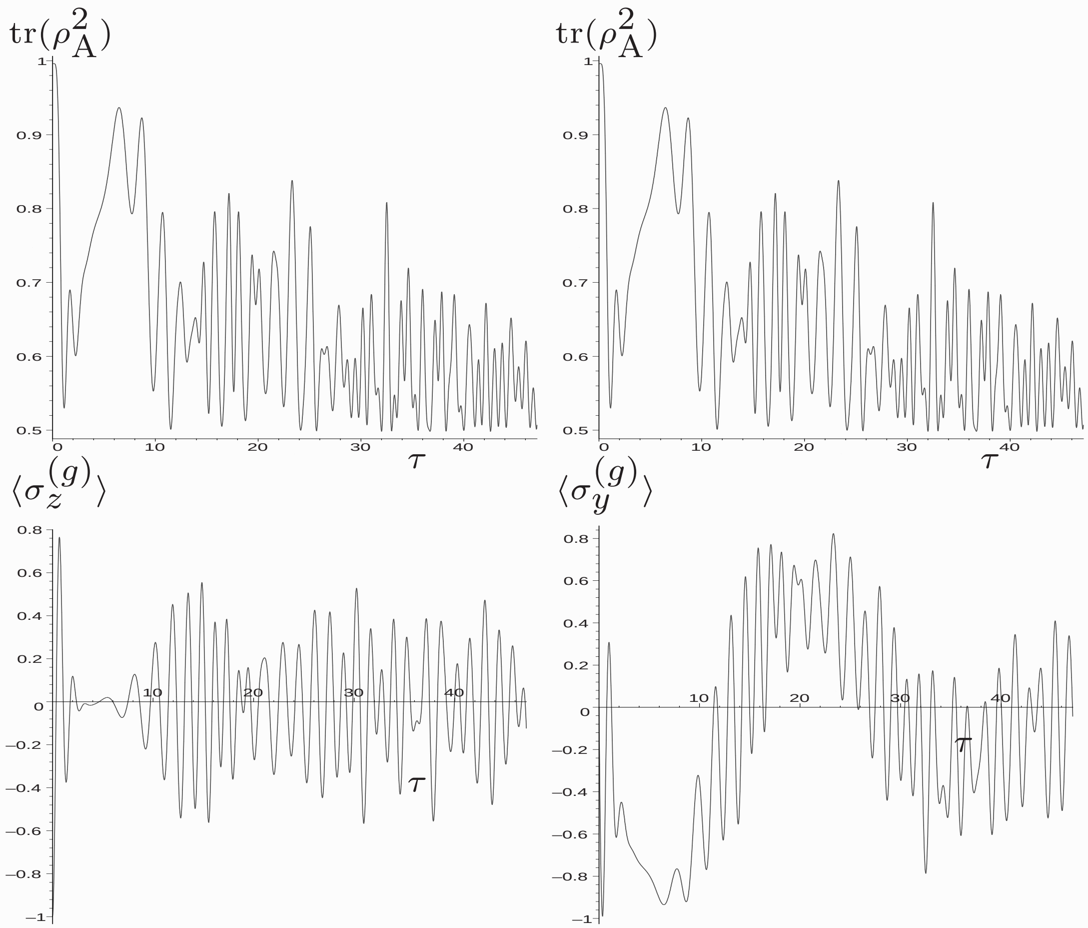
Generation of entanglement between atoms and modes

Parameter: Initial state $|g; \eta, 0\rangle$, $g_2 = 2g_1$, $\overline{n} = 25$



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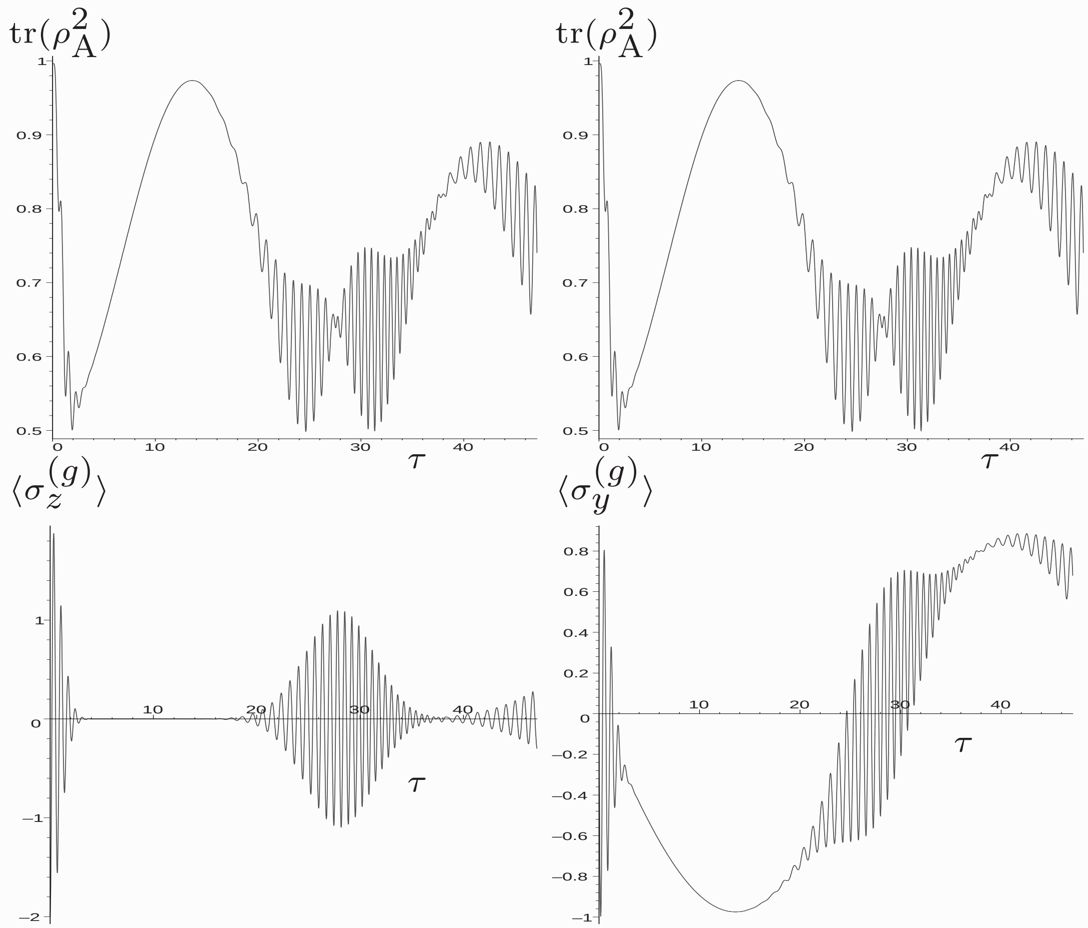

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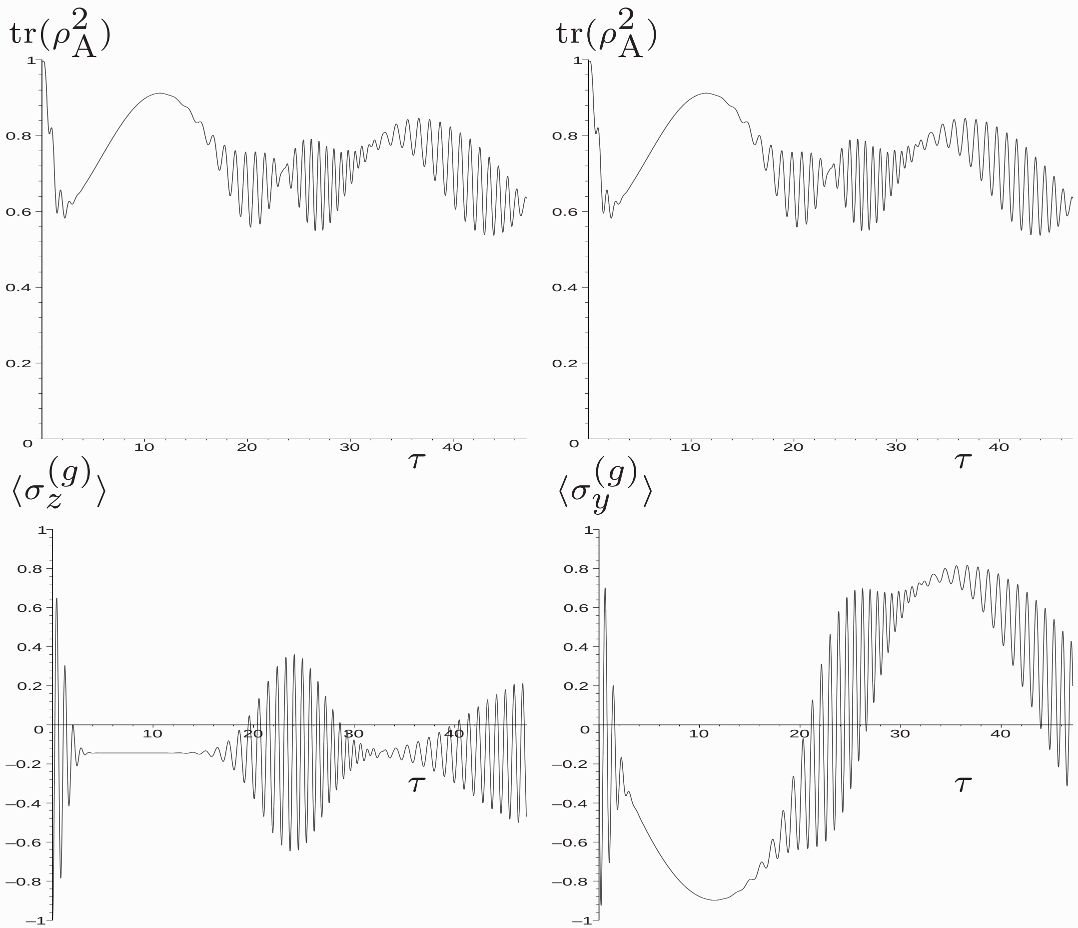
Effects of detuning on the atom-mode entanglement

Parameter: Initial state $|g;\eta,0\rangle$, $g_1 = g_2 = 25\,\mathrm{kHz}$, $\overline{n} = 25$, $\Delta/2\pi = 100\,\mathrm{kHz}$



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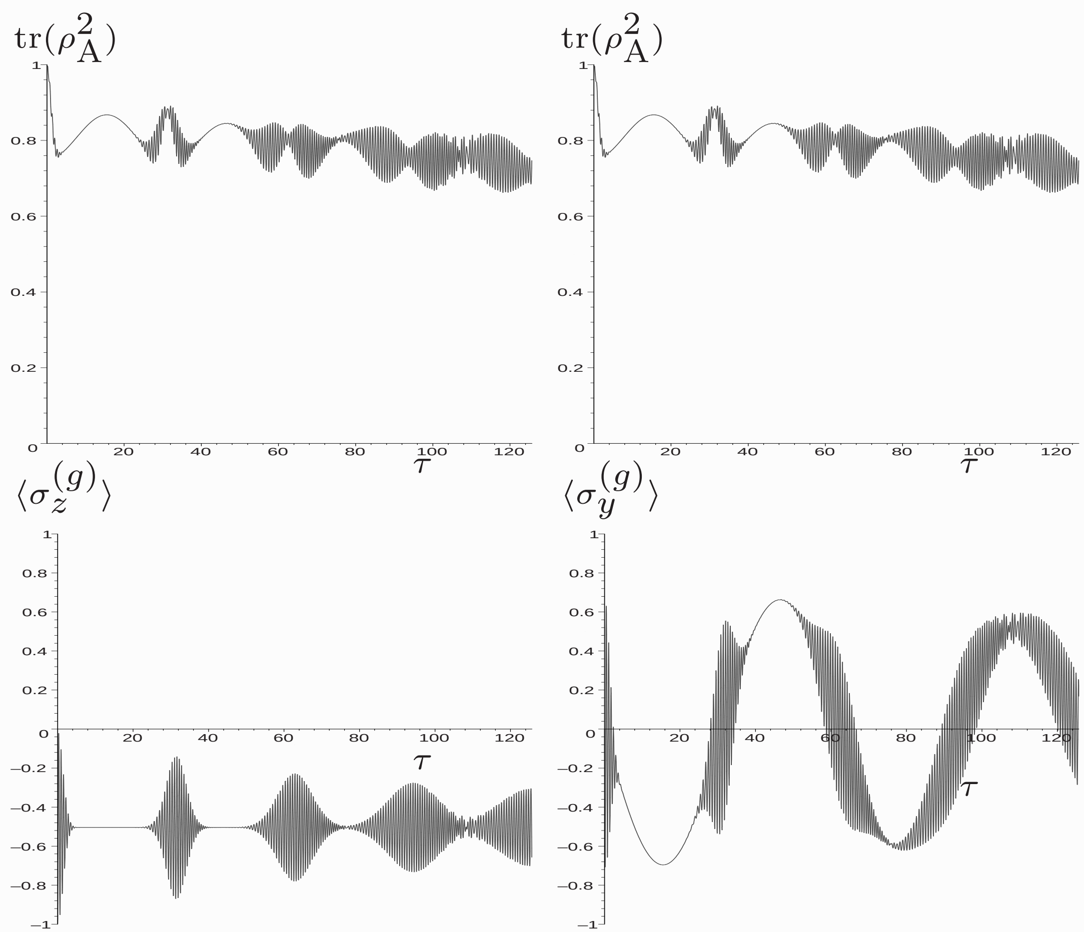
Effects of detuning on the atom-mode entanlgement

Parameter: Initial state $|g;\eta,0\rangle$, $g_1 = g_2 = 25\,\mathrm{kHz}$, $\overline{n} = 25$, $\Delta/2\pi = 250\,\mathrm{kHz}$



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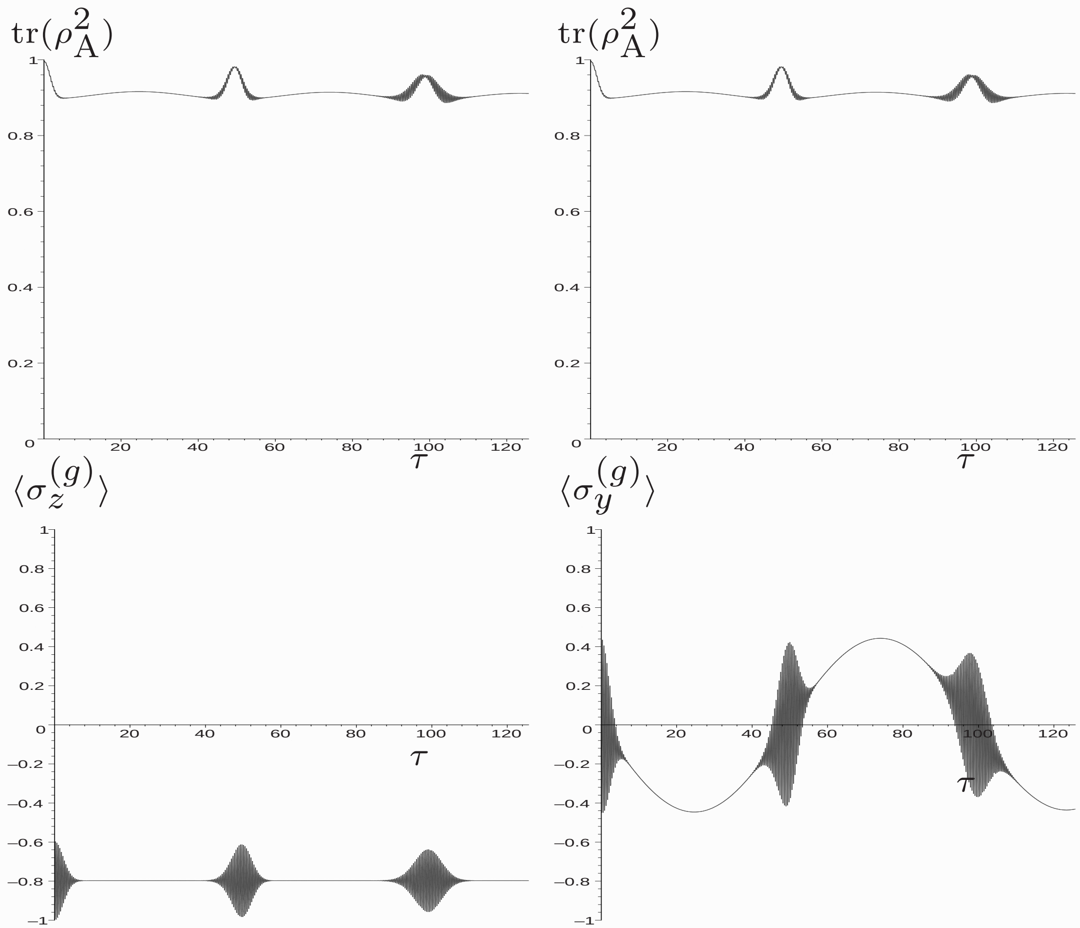
Effects of detuning on the atom-mode entanglement

Parameter: Initial state $|g;\eta,0\rangle$, $g_1 = g_2 = 25\,\mathrm{kHz}$, $\overline{n} = 25$, $\Delta/2\pi = 500\,\mathrm{kHz}$



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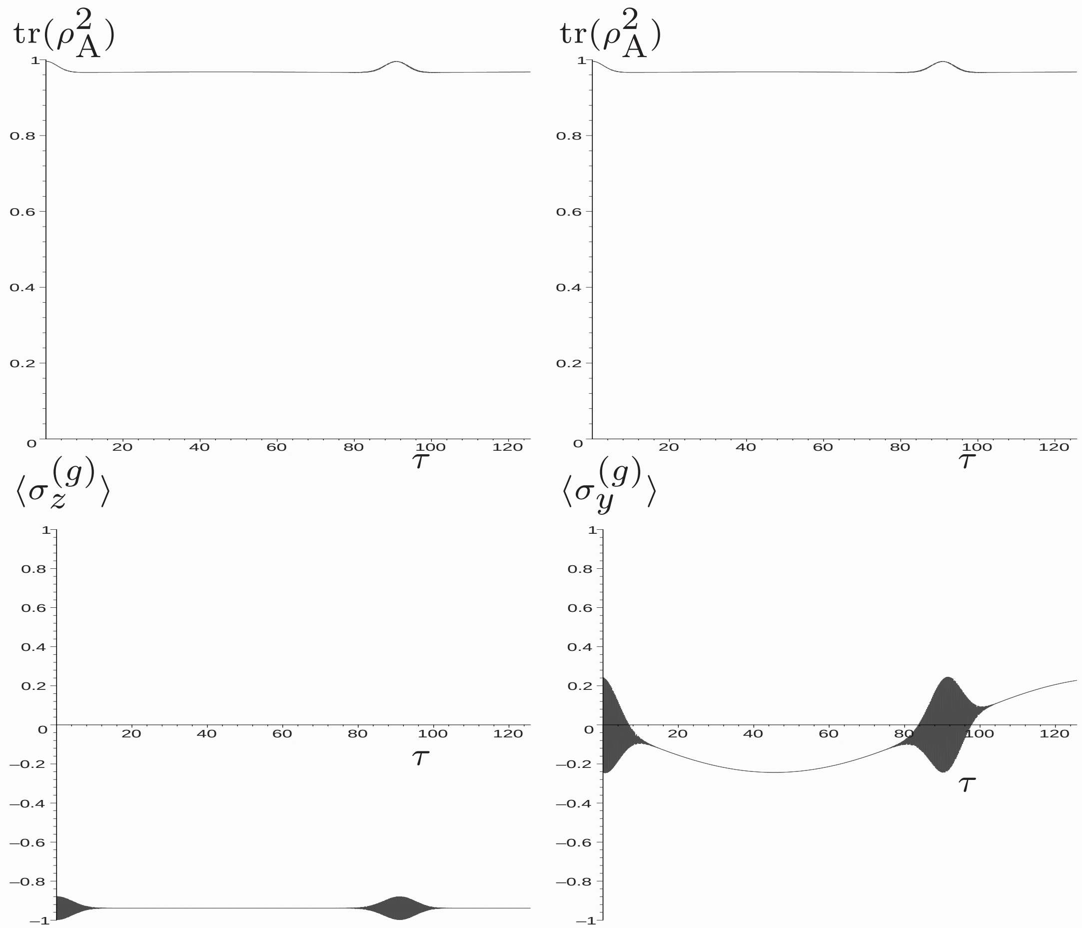
Effects of detuning on the atom-mode entanglement

Parameter: Initial state $|g;\eta,0\rangle$, $g_1 = g_2 = 25\,\mathrm{kHz}$, $\overline{n} = 25$, $\Delta/2\pi = 1\,\mathrm{MHz}$



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Generation of entangled N-photon states in a single step

$$|\Psi_N\rangle = \sum_{k=0}^{N} c_k^{(N)} |N-k, k\rangle = \sum_{m=-N/2}^{N/2} \tilde{c}_{\frac{N}{2},m} |\tfrac{N}{2}, m\rangle_{\mathrm{S}} \quad .$$

Bell-states (NOON-states) $|\Psi_N^{\pm}\rangle = \frac{1}{\sqrt{2}}(|N,0\rangle \pm |0,N\rangle) = \frac{1}{\sqrt{2}}\left(|\tfrac{N}{2},\tfrac{N}{2}\rangle_{\mathrm{S}} \pm |\tfrac{N}{2},-\tfrac{N}{2}\rangle_{\mathrm{S}}\right).$

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Generation probability for $\rho_{\Psi_N} = |\Psi_N\rangle\langle\Psi_N|$, starting from the initial state $|e;\Phi_{\mathrm{F}}\rangle = \sum_{j=0}^{\infty}{}' \sum_{m=-j}^{j} \tilde{b}_{j,m} |e; j, m\rangle_{\mathrm{S}}$, calculated from $\langle\rho_{\Psi_N}\rangle = \mathrm{tr}[\rho_{\mathrm{F}}(t)\rho_{\Psi_N}]$

$$\begin{aligned} \langle\rho_{\Psi_N}\rangle &= \left| \sum_{m=-\frac{N}{2}}^{N/2} \sum_{m'=-\frac{N}{2}}^{N/2} \tilde{b}_{\frac{N}{2},m} \tilde{c}^{*}_{\frac{N}{2},m'} C^{\frac{N}{2}}_{m',m}(\tau) \right|^2 \\ &\quad + \left| \sum_{m=-\frac{N-1}{2}}^{(N-1)/2} \sum_{m'=-\frac{N}{2}}^{N/2} \tilde{b}_{\frac{N-1}{2},m} \tilde{c}^{*}_{\frac{N}{2},m'} S^{\frac{N-1}{2}}_{m',m}(\tau) \right|^2 \quad . \end{aligned}$$

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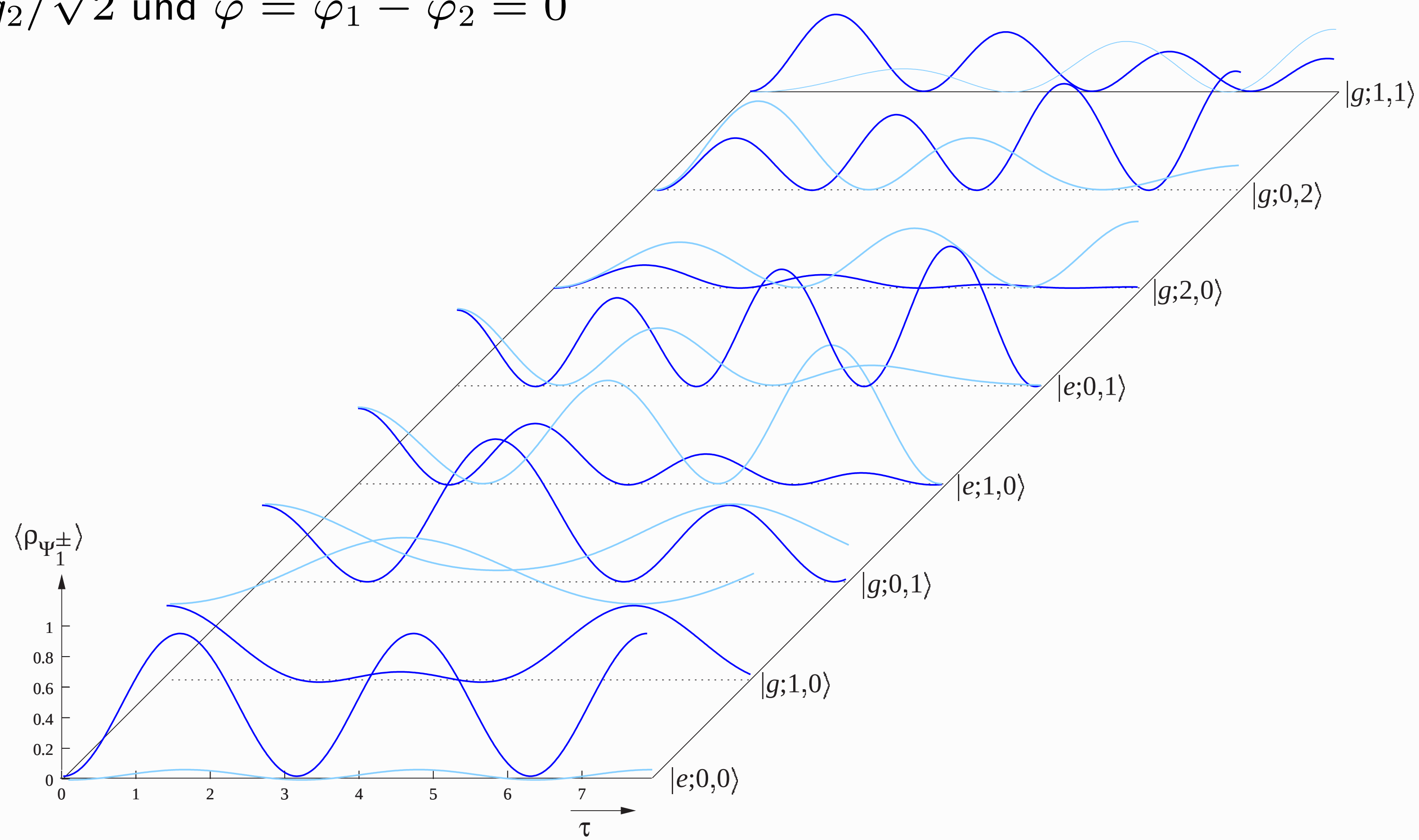
Generation of entangled N-photon field states in a single step

$$|\Psi_1^{\pm}\rangle = \frac{1}{\sqrt{2}}(|1,0\rangle \pm |0,1\rangle)$$

$g_1 = g_2/\sqrt{2}$ und $\varphi = \varphi_1 - \varphi_2 = 0$



Parametric plot of generation probabilities $\langle\rho_{\Psi_1^+}\rangle$ (dark blue) und $\langle\rho_{\Psi_1^-}\rangle$ (light blue) as function of time $\tau = gt$ for several initial states.

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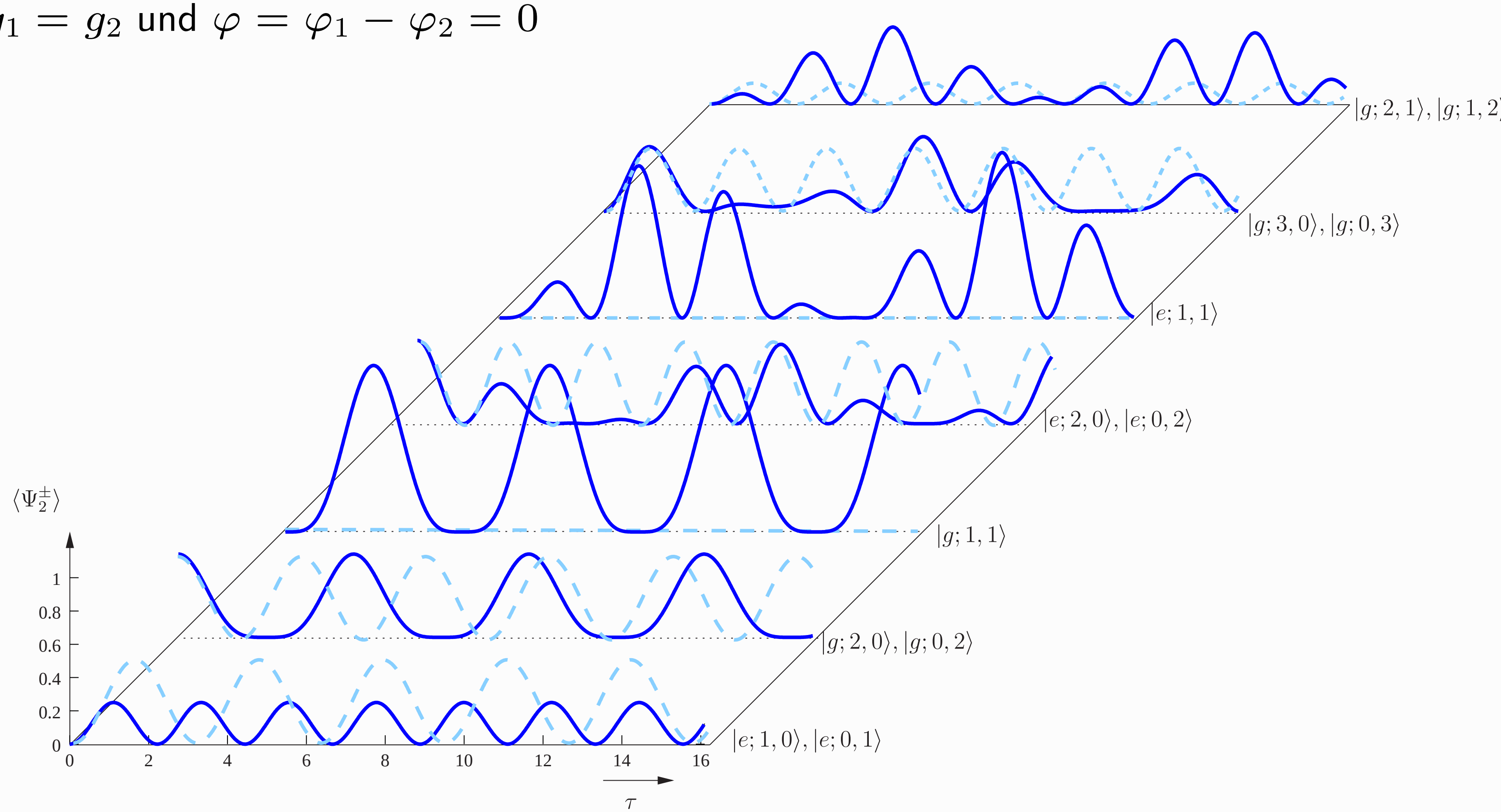
Generation of entangled N-photon field states in a single step

$$|\Psi_2^{\pm}\rangle = \frac{1}{\sqrt{2}}(|2,0\rangle \pm |0,2\rangle)$$

$g_1 = g_2$ und $\varphi = \varphi_1 - \varphi_2 = 0$



Parametric plot of generation probabilities $\langle\rho_{\Psi_2^+}\rangle$ (dark blue) und $\langle\rho_{\Psi_2^-}\rangle$ (light blue) as function of time $\tau = gt$ for several initial states.

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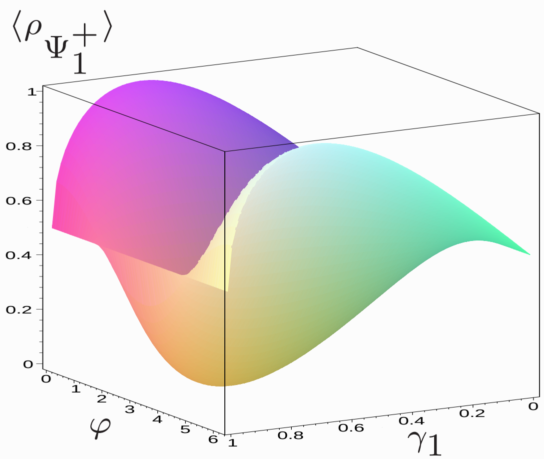
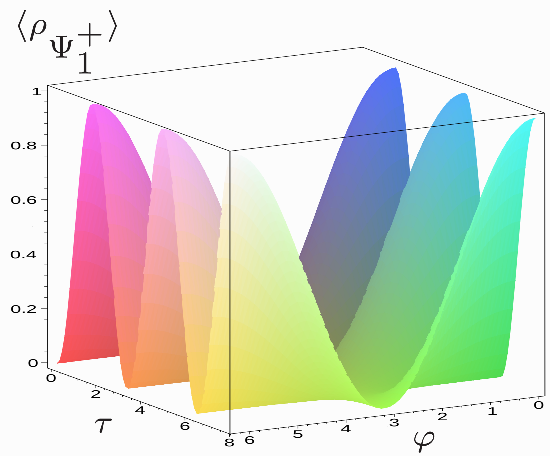
Generation of entangled N-photon field states in a single step

Initial state $|e; 0, 0\rangle$



$\tau = \pi/2$



$\gamma_1 = \gamma_2$

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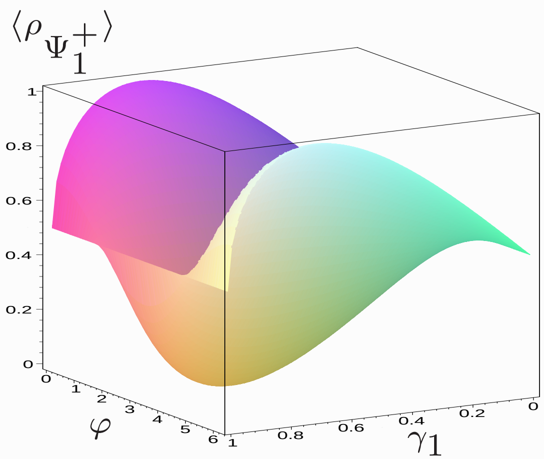
Generation of entangled N-photon field states in a single step

Initial state $|e; 0, 0\rangle$



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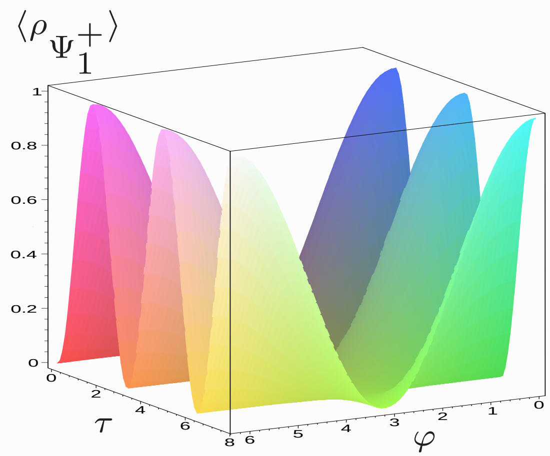

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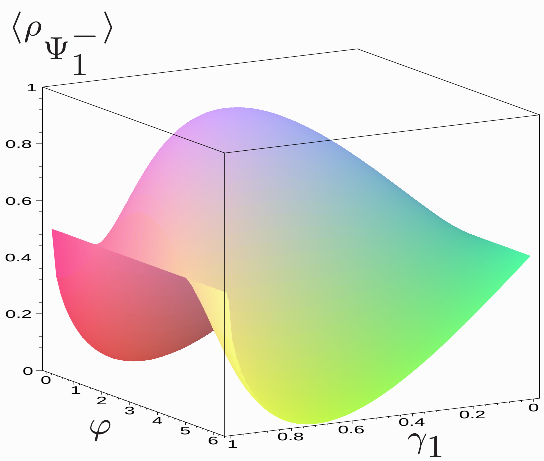

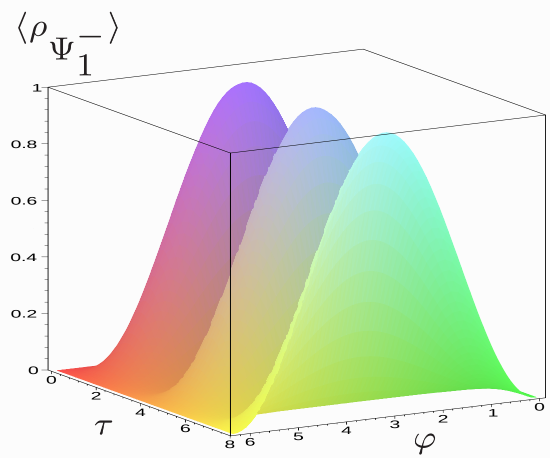

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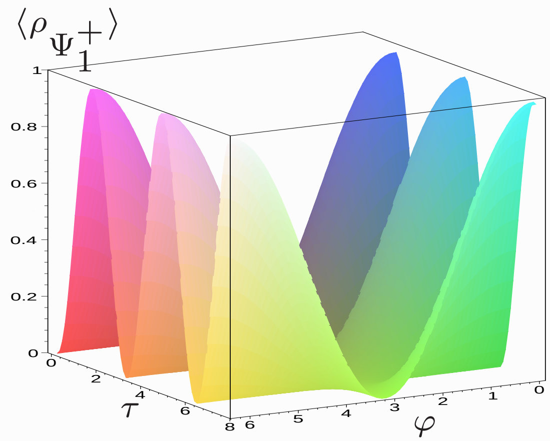
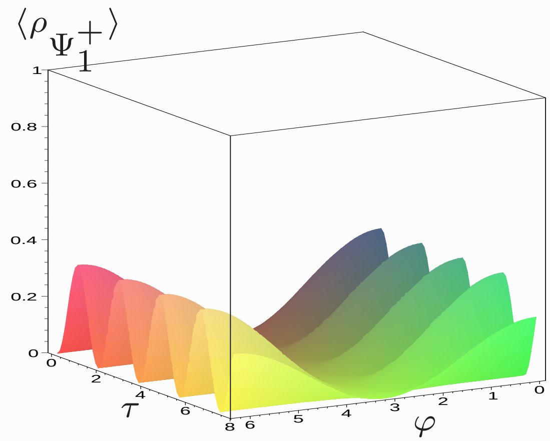
Detuning effects $\Delta \neq 0$



$|e;0,0\rangle$, $g_1 = g_2 = 25\,\mathrm{kHz}$ and $\Delta/2\pi = 10\,\mathrm{kHz}$



$\Delta/2\pi = 100\,\mathrm{kHz}$

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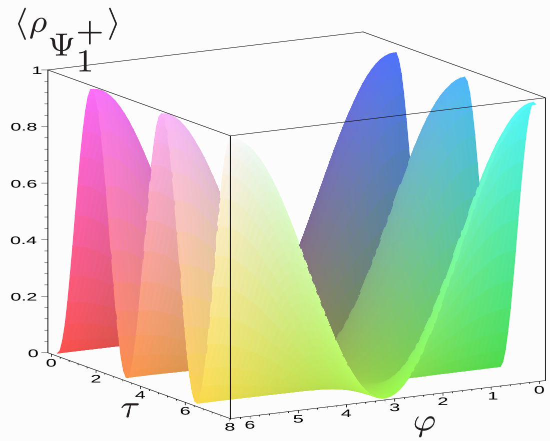
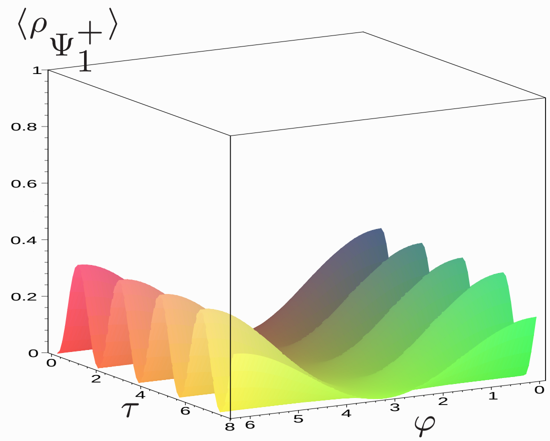
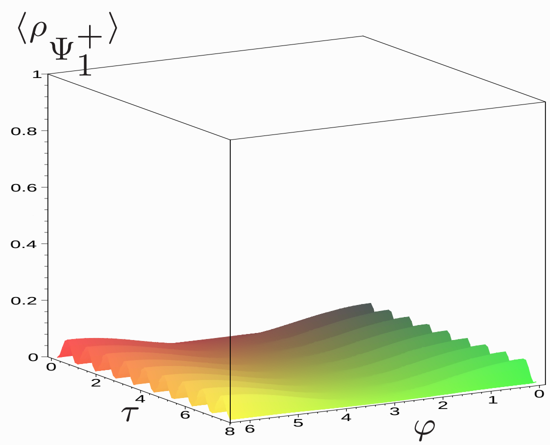
Detuning effects $\Delta \neq 0$



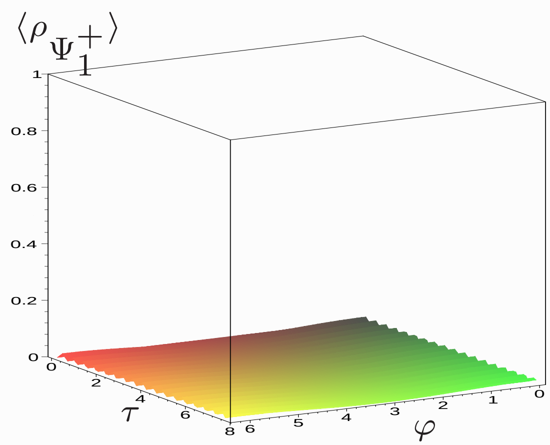
$|e;0,0\rangle$, $g_1 = g_2 = 25\,\mathrm{kHz}$ and $\Delta/2\pi = 10\,\mathrm{kHz}$



$\Delta/2\pi = 100\,\mathrm{kHz}$



$\Delta/2\pi = 250\,\mathrm{kHz}$



$\Delta/2\pi = 500\,\mathrm{kHz}$

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Conditional generation of entangled N-photon field states

- First step:

$$U(\tau_1)\,|e;0,0\rangle\!\rangle_{\mathrm{S}} = \cos\tau_1\,|e;0,0\rangle\!\rangle_{\mathrm{S}} - \mathrm{i}\sin\tau_1\,\left|g;\tfrac{1}{2},\tfrac{1}{2}\right\rangle\!\!\right\rangle_{\mathrm{S}} \quad .$$

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- Measuring the atom in the ground state $|g\rangle$

$$|\chi_1\rangle = K_1(-{\rm i})\sin\tau_1\,\left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle\!\!\right\rangle_{\rm S} \quad ,$$

leaves the field in the state $\left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle\!\!\right\rangle_{\rm S}$.

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  leaves the field in the state $\left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle\!\!\right\rangle_{\mathrm{S}}$.
- Field state after N conditional steps:

$$|\chi_N\rangle = \left|\tfrac{N}{2},\tfrac{N}{2}\right\rangle\!\!\right\rangle_{\mathrm{S}} = \sum_{k=0}^{N} D^{(\frac{N}{2})}_{\frac{N}{2}-k,\frac{N}{2}}(\varphi,\vartheta,\chi)\,|N-k,k\rangle \quad .$$

About useful entanglement see: S.J. van Enk, Phys. Rev. A **67**, 022303 (2003).

This is just the general N-photon field state

$$|\Psi_N\rangle = \sum_{k=0}^{N} c_k^{(N)}|N-k,k\rangle \quad .$$

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Non-conditional generation of entangled N-photon field states

- Reduced field density matrix after the first excited atom has interacted with the modes:

$$\rho_{\mathrm{F}}^{(1)}(\tau_1) = \cos^2\tau_1\,|0,0\rangle\rangle_{\mathrm{S}}\langle\langle 0,0| + \sin^2\tau_1\,\left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle\rangle_{\mathrm{S}}\langle\langle\tfrac{1}{2},\tfrac{1}{2}\right|$$

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- Reduced density operator after the second excited atom has interacted with the modes:

$$\begin{aligned}\rho_{\mathrm{F}}^{(2)}(\tau_1,\tau_2) = &\cos^2\tau_1 \cos^2\tau_2 \, |0,0\rangle\!\rangle_{\mathsf{S}}\langle\!\langle 0,0| \\ &+(\cos^2\tau_1 \sin^2\tau_2 + \sin^2\tau_1 \cos^2(\tau_2\sqrt{2})) \, \left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle\!\!\right\rangle_{\mathsf{S}}\left\langle\!\!\left\langle \tfrac{1}{2},\tfrac{1}{2}\right| \\ &+\sin^2\tau_1 \sin^2(\tau_2\sqrt{2}) \, |1,1\rangle\!\rangle_{\mathsf{S}}\langle\!\langle 1,1| \quad .\end{aligned}$$

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Non-conditional generation of entangled N-photon field states

- Reduced field density matrix after the first excited atom has interacted with the modes:

$$\rho_{\mathrm{F}}^{(1)}(\tau_1) = \cos^2\tau_1\,|0,0\rangle\!\rangle_{\mathsf{S}}\langle\!\langle 0,0| + \sin^2\tau_1\,\left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle\!\!\right\rangle_{\mathsf{S}}\left\langle\!\!\left\langle\tfrac{1}{2},\tfrac{1}{2}\right|\right.$$

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- Reduced density operator after n steps:

$$\rho_{\mathrm{F}}^{(n)}(\{\tau_n\}) = {\sum_{j=0}^{n/2}}' p_j^{(n)}(\{\tau_n\})\,|j,j\rangle\!\rangle_{\mathsf{S}}\langle\!\langle j,j| \quad ,$$

The coefficients are defined recursively:

$$\begin{aligned}p_0^{(n)} &= \cos^2(\tau_n)p_0^{(n-1)}, \quad p_{n/2}^{(n)} = \sin^2(\tau_n\sqrt{n})p_{(n-1)/2}^{(n-1)} \quad , \\ p_j^{(n)} &= \cos^2(\tau_n\sqrt{2j+1})p_j^{(n-1)} + \sin^2(\tau_n\sqrt{2j})p_{j-\frac{1}{2}}^{(n-1)} \quad .\end{aligned}$$

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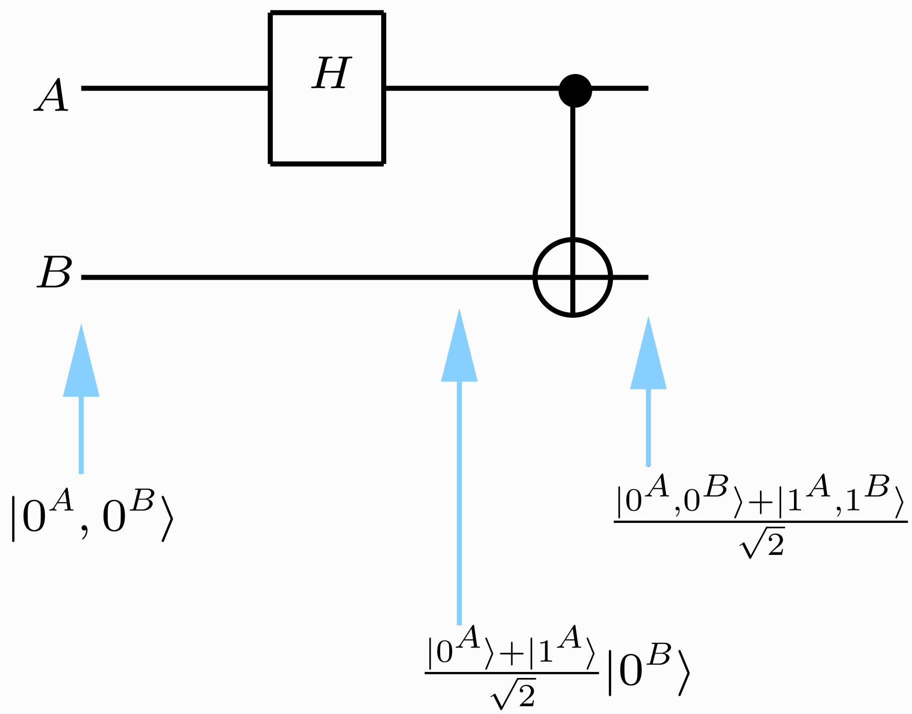
Circuits



$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad \oplus = \text{CNOT} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

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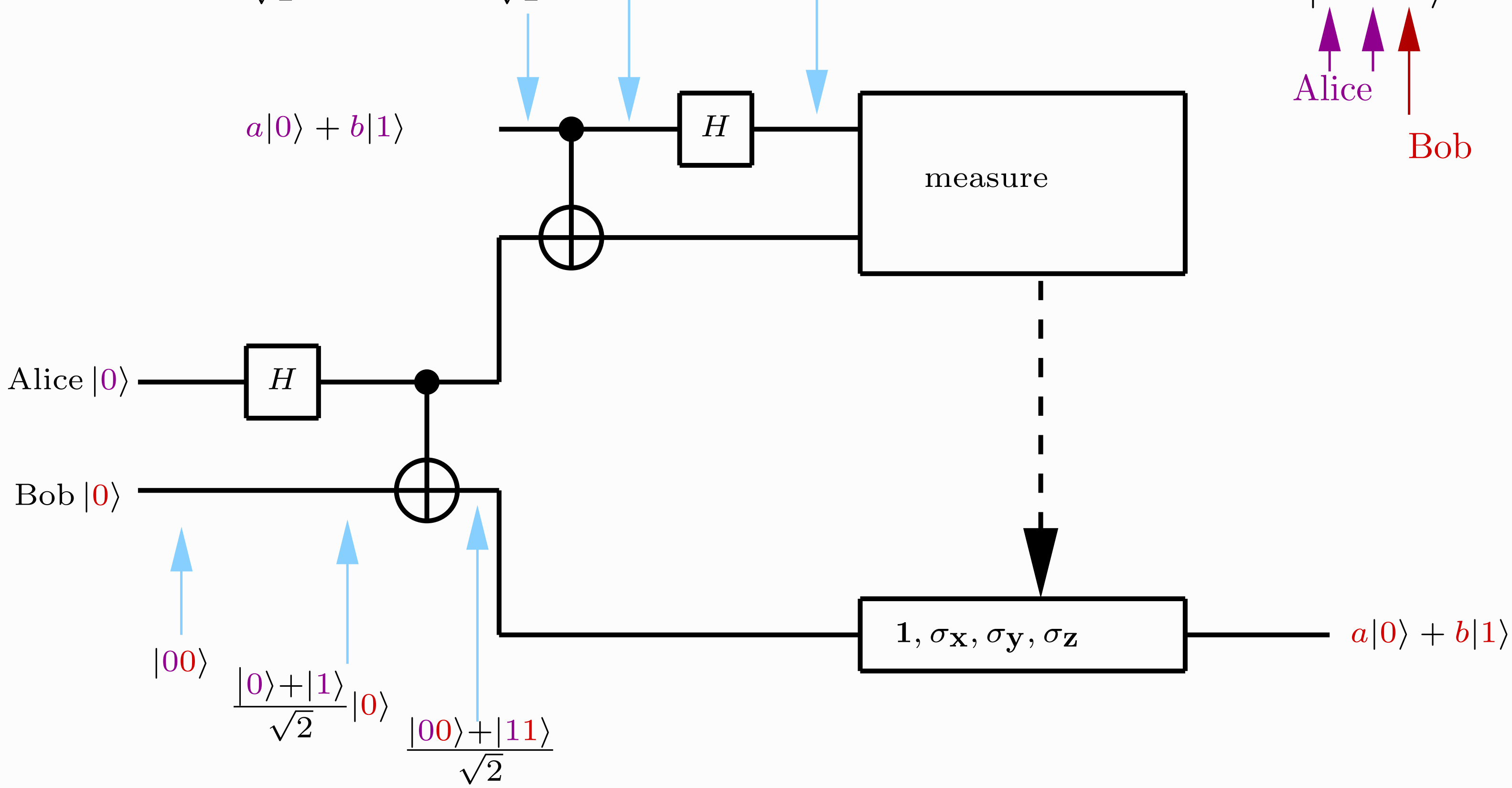
$$a\frac{|000\rangle+|100\rangle+|011\rangle+|111\rangle}{2}+b\frac{|010\rangle-|110\rangle+|001\rangle-|101\rangle}{2}$$

$$=|00\rangle\frac{a|0\rangle+b|1\rangle}{2}+|01\rangle\frac{a|1\rangle+b|0\rangle}{2}+|10\rangle\frac{a|0\rangle-b|1\rangle}{2}+|11\rangle\frac{a|1\rangle-b|0\rangle}{2}$$

$$a\frac{|000\rangle+|011\rangle}{\sqrt{2}}+b\frac{|110\rangle+|101\rangle}{\sqrt{2}}$$

$$a\frac{|000\rangle+|011\rangle}{\sqrt{2}}+b\frac{|100\rangle+|111\rangle}{\sqrt{2}}$$



Quantum Teleportation Circuit

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Summary

- The two-mode Jaynes-Cummings Model describes the interaction of single atoms with two cavity modes
- An exact solution of the model can be found in the case of $\omega_1 = \omega_2$
- We showed that the generation of maximally entangled NOON-states is possible
- Possible applications
  - Quantum Lithography
  - Quantum Teleportation

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